Name:__________________________

SM3 2.4: Synthetic Division and Remainder Thm

Vocabulary: Synthetic Division, Remainder Theorem

Synthetic division is a short-hand version of polynomial long division. The root that solves the linear divisor is drawn in a box with the coefficients of the polynomial dividend in descending order to the right of the box. The first term drops to start. Moving diagonally up-right requires multiplication by the contents of the box; moving down requires summing the column. The bottom-right corner will be the remainder. The bottom numbers directly left of the remainder are the coefficients of the quotient.

Example: Divide $\dfrac{x^2+9x+7}{x-4}$

$$\begin{array}{c|ccc} \boxed{4} & & & \\ & \downarrow & & \\ \hline & & & \boxed{} \end{array}$$

The linear divisor, $x-4$, when set $=0$ has the solution $x=4$. So we put 4 into the box.

$$\begin{array}{c|ccc} \boxed{4} & 1 & 9 & 7 \\ & \downarrow & & \\ \hline & 1 & & \boxed{} \end{array}$$

The coefficients of the polynomial dividend in descending order are placed to the right of the box. The lead coefficient drops down every time.

$$\begin{array}{c|ccc} \boxed{4} & 1 & 9 & 7 \\ & \downarrow & & \\ \hline & & & \boxed{} \end{array}$$

Fill out the empty spaces in the same order every time: down, then up-right, repeat until the remainder box is filled out.

To move down, sum the column. To move up-right, you multiply bottom number by the box.

$$\begin{array}{c|ccc} \boxed{4} & 1 & 9 & 7 \\ & \downarrow & 4 & \\ \hline & 1 & 13 & \boxed{} \end{array}$$

The 1 at the bottom is multiplied by the box. As $1 \cdot 4 = 4$, we place a 4 up-right from the 1.

The column containing 9 and 4 is summed. As $9+4=13$, we place a 13 at the bottom of the column.

$$\begin{array}{c|ccc} \boxed{4} & 1 & 9 & 7 \\ & \downarrow & 4 & \\ \hline & 1 & 13 & \boxed{} \end{array}$$

We've completed one column, now we need to do another. Calculate the numbers that belongs just below the 7 and in the remainder box.

The division is complete, it's time to interpret the result. The number in the remainder box has yet to be divided by the divisor, so we write a rational expression with the number divided by the divisor.

$$\frac{}{x-4}$$

$$ + + \frac{}{x-4}$$

The numbers to the left of the remainder box are the coefficients of the polynomial portion of the quotient.

When the remainder is 0, we say that the divisor *divides* the dividend. When the remainder is not 0, we say that the divisor does not divide the dividend. A basic example: 3 divides 12; 3 does not divide 17. A polynomial example: $x-2$ divides x^2-4; $x-2$ does not divide x^2-5.

The Remainder Theorem

If $f(x)$ is a polynomial, $f(x)=x^n+x^{n-1}+\cdots$, then the remainder of $\frac{f(x)}{x-a}$ is equal to $f(a)$.

Example: On the previous page, we divided: $\frac{x^2+9x+7}{x-4}$ and found that:

$$\frac{x^2+9x+7}{x-4}=x+13+\frac{59}{x-4}$$

Multiplying by $(x-4)$ on both sides gives:

$$x^2+9x+7=(x+13)(x-4)+59$$

If we evaluate at $x=4$:

$$4^2+9(4)+7=(4+13)(4-4)+59$$
$$4^2+9(4)+7=(17)(0)+59$$
$$4^2+9(4)+7=59$$

$f(4)$ is equal to the remainder we got from dividing by $(x-4)$.

Example: Prove whether $(x+5)$ is a factor of $x^2+2x-35$ and write a sentence explaining your reasoning.

$$(-5)^2+2(-5)-35$$
$$25-10-35$$
$$-20$$

The value of $x^2+2x-35$, evaluated at $x=-5$ is not 0. Therefore, $(x+5)$ is not a factor of $x^2+2x-35$.

Vocabulary Problems: Interpret the results of the synthetic division problem by rewriting the problem as $\frac{dividend}{divisor} = quotient.$

1)

$\boxed{2}$	3	5	7
	↓	6	22
	3	11	$\boxed{29}$

$\underline{\hspace{3cm}} =$

2)

$\boxed{-7}$	1	2	4
	↓	−7	−35
	1	−5	$\boxed{-31}$

$\underline{\hspace{3cm}} =$

3) What is the remainder of $\frac{4x-11}{x-6}$?

4) What is the remainder of $\frac{5x^3-2x^2+4x-9}{x+3}$?

Problems: Simplify each expression into a quotient that contains a polynomial and $\frac{remainder}{divisor}$.

5) $\dfrac{x^2+10x+24}{x-2}$

6) $\dfrac{x^2-3x+7}{x-4}$

7) $\dfrac{x^2+7x-12}{x-3}$

8) $\dfrac{x^2+x-30}{x+6}$

9) $\dfrac{x^2+11x+8}{x+1}$

10) $\dfrac{x^2+4x-5}{x}$

11) $\dfrac{3x^2+x+6}{x-8}$

12) $\dfrac{-4x^2-3x-1}{x+6}$

13) $\dfrac{2x^2-12}{x-5}$

14) $\dfrac{x^3 - 5x^2 + 10x - 50}{x - 5}$

15) $\dfrac{x^3 + x^2 + x + 1}{x - 1}$

16) $\dfrac{2x^3 + 3x^2 + 4x - 5}{x + 3}$

17) $\dfrac{x^5 - 4x^4 - 2x^3 - 6x^2 + 9x + 3}{x - 2}$

18) $\dfrac{7 - x^3 - 3x^4 + x^2 - 3x^5}{x + 3}$

19) Prove whether $(x + 5)$ is a factor of $x^2 + 2x - 35$ and write a sentence explaining your reasoning.

20) Prove whether $(x - 2)$ is a factor of $x^3 - 3x^2 + 4$ and write a sentence explaining your reasoning.

21) Given $f(x) = 4x^5 - 2x^3 + 17x^2 + 4$, find $f(-3)$.

22) Given $f(x) = -7x^3 + 2x^2 + 15x - 26$, find $f(13)$.